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Constant moving crack in a magnetoelectroelastic material under anti-plane shear loading

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Abstract

Analytical solutions for an anti-plane Griffith moving crack inside an infinite magnetoelectroelastic medium under the conditions of permeable crack faces are formulated using integral transform method. The far-field anti-plane mechanical shear and in-plane electrical and magnetic loadings are applied to the magnetoelectroelastic material. Expressions for stresses, electric displacements and magnetic inductions in the vicinity of the crack tip are derived. Field intensity factors for magnetoelectroelastic material are obtained. The stresses, electric displacements and magnetic inductions at the crack tip show inverse square root singularities. The moving speed of the crack have influence on the dynamic electric displacement intensity factor (DEDIF) and the dynamic magnetic induction intensity factor (DMIIF), while the dynamic stress intensity factor (DSIF) does not depend on the velocity of the moving crack. When the crack is moving at very lower or very higher speeds, the crack will propagate along its original plane; while in the range of $M_{c1} < M < M_{c2}$, the propagation of the crack possibly brings about the branch phenomena in magnetoelectroelastic media.

Keywords: Moving crack; Magnetoelectroelastic medium; Integral transform method; Permeable; Branch phenomena

1. Introduction

Fibrous and laminated composites made of piezoelectric–piezomagnetic materials exhibit magnetoelectric effect that is not present in single-phase piezoelectric or piezomagnetic materials, and have found increasingly wide engineering applications, particularly in the aerospace and automotive industries. Numerous investigators have carried out studies on the properties of piezoelectric/piezomagnetic composites in

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recent years (see e.g., Nan, 1994; Huang et al., 2000; Li, 2000; Pan, 2001; Wang and Shen, 2002; Wang and Zhong, 2003). In particular, the damage tolerance and reliability for the composites have been matters of concern, and there is a growing interest among researchers in solving fracture mechanic problems in media possessing coupled piezoelectric, piezomagnetic and magnetoelectric effects, this is, magnetoelectroelastic effects. Recently, Liu et al. (2001) investigated magnetoelectroelastic materials involving a cavity or a crack by including the electric field effects. Gao et al. (2003) presented an exact treatment on the crack problems in a magnetoelectroelastic solid subjected to far-field loadings. Song and Sih (2003) analyzed the crack initiation behavior in magnetoelectroelastic composite under in-plane deformation. The anti-plane crack problems in magnetoelectroelastic materials have been considered by Spyropoulos et al. (2003) and Wang and Mai (2004). To the best of the authors' knowledge, the problem of a moving crack in magnetoelectroelastic materials has not been resolved.

The objective of this paper is to seek the solution to the Yoffe-type moving crack problem in a magnetoelectroelastic material under anti-plane mechanical shear and in-plane electrical and magnetic loadings. Fourier transforms are used to reduce the problem to the solution of dual integral equations. The solution of the dual integral equations is then expressed analytically. Closed-form expressions for crack-tip fields and field intensity factors are obtained. The results indicate that the crack moving velocity will exert a significant effect on the crack-tip fields.

2. Basic equations for magnetoelectroelastic media

We consider a linear magnetoelectroelastic solid and denote the rectangular coordinates of a point by $x_j (j = 1, 2, 3)$. Dynamic equilibrium equations are given as

$$\sigma_{ij,i} + f_i = \rho \frac{\partial^2 u_j}{\partial t^2}, \quad D_{i,i} - f_e = 0, \quad B_{i,i} - f_m = 0, \tag{1}$$

where σ_{ij}, D_i and B_i are components of stress, electrical displacement and magnetic induction, respectively; f_j, f_e and f_m are the body force, electric charge density and electric current density, respectively; ρ is the mass density of the magnetoelectroelastic material; a comma followed by $i (i = 1, 2, 3)$ denotes partial differentiation with respect to the coordinate x_i, and the usual summation convention over repeated indices is applied. Constitutive equations can be written as

$$\begin{aligned} \sigma_{ij} &= c_{ijks}\varepsilon_{ks} - e_{sij}E_s - h_{sij}H_s, \\ D_i &= e_{iks}\varepsilon_{ks} + \lambda_{is}E_s + \beta_{is}H_s, \\ B_i &= h_{iks}\varepsilon_{ks} + \beta_{is}E_s + \gamma_{is}H_s, \end{aligned} \tag{2}$$

where ε_{ks}, E_s and H_s are components of strain, electric field and magnetic field, respectively; c_{ijks}, e_{iks}, h_{iks} and β_{is} are elastic, piezoelectric, piezomagnetic and electromagnetic constants, respectively; λ_{is} and γ_{is} are dielectric permittivities and magnetic permeabilities, respectively. The following reciprocal symmetries hold:

$$\begin{aligned} &c_{ijks} = c_{jiks} = c_{ijsk} = c_{ksij}, \quad e_{sij} = e_{sji}, \\ &h_{sij} = h_{sji}, \quad \beta_{ij} = \beta_{ji}, \quad \lambda_{ij} = \lambda_{ji}, \quad \gamma_{ij} = \gamma_{ji}. \end{aligned} \tag{3}$$

Gradient equations are

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad E_i = -\phi_{,i}, \quad H_i = -\varphi_{,i}, \tag{4}$$

where u_i is the displacement vector, ϕ and $\varphi$are electric potential and magnetic potential, respectively.

For a special case of a transversely isotropic magnetoelectroelastic medium with x_3 as a symmetry axis, the constitutive equations (2) take the form as follows (Pan, 2001):

$$\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{31}\\ \sigma_{12}\end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0\\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0\\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & c_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & c_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & c_{66}\end{pmatrix}\begin{pmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{23}\\ 2\varepsilon_{31}\\ 2\varepsilon_{12}\end{pmatrix} - \begin{pmatrix} 0 & 0 & e_{31}\\ 0 & 0 & e_{31}\\ 0 & 0 & e_{33}\\ 0 & e_{15} & 0\\ e_{15} & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}E_1\\ E_2\\ E_3\end{pmatrix} - \begin{pmatrix} 0 & 0 & h_{31}\\ 0 & 0 & h_{31}\\ 0 & 0 & h_{33}\\ 0 & h_{15} & 0\\ h_{15} & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}H_1\\ H_2\\ H_3\end{pmatrix}, \tag{5}$$

$$\begin{pmatrix}D_1\\ D_2\\ D_3\end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & e_{15} & 0\\ 0 & 0 & 0 & e_{15} & 0 & 0\\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{23}\\ 2\varepsilon_{31}\\ 2\varepsilon_{12}\end{pmatrix} + \begin{pmatrix}\lambda_{11} & 0 & 0\\ 0 & \lambda_{11} & 0\\ 0 & 0 & \lambda_{33}\end{pmatrix}\begin{pmatrix}E_1\\ E_2\\ E_3\end{pmatrix} + \begin{pmatrix}\beta_{11} & 0 & 0\\ 0 & \beta_{11} & 0\\ 0 & 0 & \beta_{33}\end{pmatrix}\begin{pmatrix}H_1\\ H_2\\ H_3\end{pmatrix}, \tag{6}$$

$$\begin{pmatrix}B_1\\ B_2\\ B_3\end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & h_{15} & 0\\ 0 & 0 & 0 & h_{15} & 0 & 0\\ h_{31} & h_{31} & h_{33} & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{23}\\ 2\varepsilon_{31}\\ 2\varepsilon_{12}\end{pmatrix} + \begin{pmatrix}\beta_{11} & 0 & 0\\ 0 & \beta_{11} & 0\\ 0 & 0 & \beta_{33}\end{pmatrix}\begin{pmatrix}E_1\\ E_2\\ E_3\end{pmatrix} + \begin{pmatrix}\gamma_{11} & 0 & 0\\ 0 & \gamma_{11} & 0\\ 0 & 0 & \gamma_{33}\end{pmatrix}\begin{pmatrix}H_1\\ H_2\\ H_3\end{pmatrix}, \tag{7}$$

where $c_{66} = (c_{11} - c_{12})/2$. The governing equations simplify considerably if we consider only the out-of-plane displacement, the in-plane electric fields and in-plane magnetic fields, i.e.,

$$u_1 = u_2 = 0, \quad u_3 = w(x,y), \tag{8}$$

$$E_1 = E_x(x,y), \quad E_2 = E_y(x,y), \quad E_3 = 0, \tag{9}$$

$$H_1 = H_x(x,y), \quad H_2 = H_y(x,y), \quad H_3 = 0. \tag{10}$$

In this case, if there is no body force, electric charge density and electric current density, the governing equations (1) simplify to

$$\begin{aligned} &c_{44}\nabla^2 w + e_{15}\nabla^2\phi + h_{15}\nabla^2\varphi = \rho\frac{\partial^2 w}{\partial t^2}, \\ &e_{15}\nabla^2 w - \lambda_{11}\nabla^2\phi - \beta_{11}\nabla^2\varphi = 0, \\ &h_{15}\nabla^2 w - \beta_{11}\nabla^2\phi - \gamma_{11}\nabla^2\varphi = 0, \end{aligned} \tag{11}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the two-dimensional Laplace operator in the variables x and y, and the constitutive relations (2), (5)–(7) become

$$\begin{pmatrix}\sigma_{zy}\\ D_y\\ B_y\end{pmatrix} = \begin{pmatrix} c_{44} & e_{15} & h_{15}\\ e_{15} & -\lambda_{11} & -\beta_{11}\\ h_{15} & -\beta_{11} & -\gamma_{11}\end{pmatrix}\begin{pmatrix}\frac{\partial w}{\partial y}\\ \frac{\partial \phi}{\partial y}\\ \frac{\partial \varphi}{\partial y}\end{pmatrix}, \quad \begin{pmatrix}\sigma_{zx}\\ D_x\\ B_x\end{pmatrix} = \begin{pmatrix} c_{44} & e_{15} & h_{15}\\ e_{15} & -\lambda_{11} & -\beta_{11}\\ h_{15} & -\beta_{11} & -\gamma_{11}\end{pmatrix}\begin{pmatrix}\frac{\partial w}{\partial x}\\ \frac{\partial \phi}{\partial x}\\ \frac{\partial \varphi}{\partial x}\end{pmatrix}. \tag{12}$$

Introducing two new functions Φ and Ψ as

$$\Phi = \phi + m\cdot w, \quad \Psi = \varphi + n\cdot w, \tag{13}$$

where

$$m = \frac{\beta_{11}h_{15} - \gamma_{11}e_{15}}{\lambda_{11}\gamma_{11} - \beta_{11}^2}, \quad n = \frac{\beta_{11}e_{15} - \lambda_{11}h_{15}}{\lambda_{11}\gamma_{11} - \beta_{11}^2}. \tag{14}$$

Eq. (11) become

$$\nabla^2 w = \frac{1}{C^2}\frac{\partial^2 w}{\partial t^2}, \quad \nabla^2\Phi = 0, \quad \nabla^2\Psi = 0, \tag{15}$$

where

$$C = \sqrt{\frac{\mu}{\rho}}, \quad \mu = c_{44} + \frac{\gamma_{11}e_{15}^2 + \lambda_{11}h_{15}^2 - 2\beta_{11}e_{15}h_{15}}{\lambda_{11}\gamma_{11} - \beta_{11}^2}, \tag{16}$$

and C, μ and ρ are the speed of the magnetoelectroelastic shear wave, the magnetoelectroelastic constant, and the material density, respectively.

3. Problem statement and method of solution

Consider a Griffith crack of length $2c$ moving at constant speed v in an infinite magnetoelectroelastic material, which is subjected to far-field mechanical, electrical and magnetic loads as shown in Fig. 1. This type of crack is the so-called Yoffe-type moving crack (Yoffe, 1951; Chen and Yu, 1997; Chen et al., 1998; Hou et al., 2001; Kwon and Lee, 2001, 2003; Kwon et al., 2002; Kwon, 2004).

For convenience, let a coordinate system (x, y, z) be attached to the moving crack and when $t = 0$ it coincides with the fixed coordinate system (X, Y, Z). Since the problem is in a steady state, the Galilean transformation can be introduced, i.e.

$$x = X - vt, \quad y = Y, \quad z = Z. \tag{17}$$

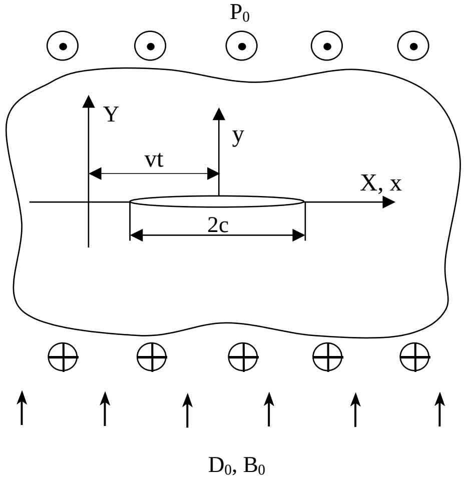

Fig. 1. A crack moving in magnetoelectroelastic material under far-field mechanical, electrical and magnetic loads.

With reference to the moving coordinates system, Eq. (15) become independent of the time variable t and may be rewritten as

$$k\frac{\partial^2 w(x,y)}{\partial x^2}+\frac{\partial^2 w(x,y)}{\partial y^2}=0,\quad \nabla^2\Phi(x,y)=0,\quad \nabla^2\Psi(x,y)=0, \tag{18}$$

where

$$k=1-(v/C)^2. \tag{19}$$

The poled magnetoelectroelastic medium is thick enough in the z-direction to allow a state of anti-plane shear, and the crack is situated along the plane ($-c<x<c$, $y=0$). Due to the assumed symmetry in geometry and loading, it is sufficient to consider the problem for $0\leqslant x<\infty$, $0\leqslant y\leqslant\infty$ only.

We will consider the boundary conditions at infinity as

$$\sigma_{yz}=P_0,\quad D_y=D_0,\quad B_y=B_0\quad (x^2+y^2\to\infty). \tag{20}$$

Fourier transforms are applied to Eq. (18), and by using the conditions in Eq. (13), the results can be obtained as follows:

$$w(x,y)=2\int_0^\infty A(\xi)\exp(-\sqrt{k}\xi y)\cos(\xi x)\,\mathrm{d}\xi+a_0y, \tag{21}$$

$$\phi(x,y)=2\int_0^\infty [B(\xi)\exp(-\xi y)-mA(\xi)\exp(-\sqrt{k}\xi y)]\cos(\xi x)\,\mathrm{d}\xi+b_0y, \tag{22}$$

$$\varphi(x,y)=2\int_0^\infty [C(\xi)\exp(-\xi y)-nA(\xi)\exp(-\sqrt{k}\xi y)]\cos(\xi x)\,\mathrm{d}\xi+c_0y, \tag{23}$$

where $A(\xi)$, $B(\xi)$ and $C(\xi)$ are the unknowns to be solved and a_0, b_0, c_0 are real constants, which will be determined from the far-field loading conditions. A simple calculation leads to the stress, electric displacement and magnetic induction expressions:

$$\sigma_{zy} = -2\int_0^\infty \xi \left\{ \begin{array}{c} (c_{44} - e_{15}m - h_{15}n)\sqrt{k}\exp(-\sqrt{k}\xi y) \\ +[e_{15}B(\xi) + h_{15}C(\xi)]\exp(-\xi y) \end{array} \right\} \cos(\xi x) d\xi + P_0, \tag{24}$$

$$D_y = 2\int_0^\infty \xi[\lambda_{11}B(\xi) + \beta_{11}C(\xi)]\exp(-\xi y)\cos(\xi x)\,\mathrm{d}\xi + D_0, \tag{25}$$

$$B_y = 2\int_0^\infty \xi[\beta_{11}B(\xi) + \gamma_{11}C(\xi)]\exp(-\xi y)\cos(\xi x)\,\mathrm{d}\xi + B_0. \tag{26}$$

The constants a_0, b_0 and c_0 can be obtained by considering the far-field loading conditions as follows:

$$\begin{pmatrix} a_0 \\ b_0 \\ c_0 \end{pmatrix} = \begin{pmatrix} c_{44} & e_{15} & h_{15} \\ e_{15} & -\lambda_{11} & -\beta_{11} \\ h_{15} & -\beta_{11} & -\gamma_{11} \end{pmatrix}^{-1} \begin{pmatrix} P_0 \\ D_0 \\ B_0 \end{pmatrix}, \tag{27}$$

The mechanical conditions for the crack case are:

$$\begin{aligned} &\sigma_{zy}(x,0) = 0, \quad (0 \leqslant x < c), \\ &w(x,0) = 0, \quad (c \leqslant x < \infty). \end{aligned} \tag{28}$$

The electrical and magnetic conditions for the permeable crack case can be expressed as (Parton and Kudryurtsev, 1998; Gao et al., 2003):

$$\begin{aligned} &D_y(x,0^+) = D_y(x,0^-), \quad E_x(x,0^+) = E_x(x,0^-), \qquad (0 \leqslant x < c), \\ &\phi(x,0) = 0, \qquad (c \leqslant x < \infty), \end{aligned} \tag{29}$$

$$\begin{aligned} &B_y(x,0^+) = B_y(x,0^-), \quad H_x(x,0^+) = H_x(x,0^-), \qquad (0 \leqslant x < c) \\ &\varphi(x,0) = 0, \qquad (c \leqslant x < \infty). \end{aligned} \tag{30}$$

The stresses, the strains, the electric field intensities, the electric displacements, the magnetic field intensities and the magnetic inductions can be obtained by making use of Eqs. (4), (12), (24)–(26).

Satisfaction of the three mixed boundary conditions (28)–(30) leads to the simultaneous dual integral equations of the following form:

$$\int_0^\infty \xi A(\xi)\cos(\xi x)\,\mathrm{d}\xi = \frac{P_0}{2\left[c_{44}\sqrt{k} + (1-\sqrt{k})(e_{15}m + h_{15}n)\right]} = \frac{R_1}{2}, \quad (0 \leqslant x < c), \tag{31}$$

$$\int_0^\infty A(\xi)\cos(\xi x)\,\mathrm{d}\xi = 0, \quad (x \geqslant c), \tag{32}$$

$$B(\xi) = mA(\xi), \quad C(\xi) = nA(\xi). \tag{33}$$

Obviously, we can get the analytical solutions of the simultaneous dual integral equations above mentioned as (Fan, 1978):

$$A(\xi) = \frac{R_1}{2}c\xi^{-1}J_1(\xi c), \quad B(\xi) = mA(\xi), \quad C(\xi) = nA(\xi), \tag{34}$$

in which $J_1()$ denotes the first order Bessel function of the first kind.

Substituting Eq. (34) into Eqs. (4), (12), (21)–(23) and following the procedure given by Fan (1978), we arrive at:

$$\begin{aligned}\sigma_{zy}+\mathrm{i}\sigma_{zx}&=(c_{44}-e_{15}m-h_{15}n)R_1\left\{\sqrt{k}\,\mathrm{Re}\left[\frac{z_v}{\sqrt{z_v^2-c^2}}-1\right]+\mathrm{i}\,\mathrm{Im}\left[\frac{z_v}{\sqrt{z_v^2-c^2}}\right]\right\}\\&\quad+(e_{15}m+h_{15}n)R_1\left[\frac{z}{\sqrt{z^2-c^2}}-1\right]+P_0,\\D_y+\mathrm{i}D_x&=-(\lambda_{11}m+\beta_{11}n)R_1\left[\frac{z}{\sqrt{z^2-c^2}}-1\right]+D_0,\\B_y+\mathrm{i}B_x&=-(\beta_{11}m+\gamma_{11}n)R_1\left[\frac{z}{\sqrt{z^2-c^2}}-1\right]+B_0,\end{aligned}\tag{35}$$

where Re and Im denote the real and imaginary parts of a complex variable respectively, $z=x+\mathrm{i}y$, $z_v=x+\mathrm{i}\sqrt{k}y$, and $\mathrm{i}=\sqrt{-1}$.

4. Field intensity factors

Evaluating the solution (35) near the right crack tip and extend the traditional concept of stress intensity factor to other field variables, we can get the singular parts of the stresses, the electric displacements and the magnetic inductions as

$$\begin{aligned}\sigma_{zy}&=\frac{K^T(v)}{\sqrt{2r_1}}\left[(1+q)\sqrt{\frac{r_1}{\tilde{r}_1}}\cos\left(\frac{\tilde{\theta}_1}{2}\right)-q\cos\left(\frac{\theta_1}{2}\right)\right],\\\sigma_{zx}&=-\frac{K^T(v)}{\sqrt{2r_1}}\left[(1+q)\sqrt{\frac{r_1}{k\tilde{r}_1}}\sin\left(\frac{\tilde{\theta}_1}{2}\right)-q\sin\left(\frac{\theta_1}{2}\right)\right],\end{aligned}\tag{36}$$

$$D_y=\frac{K^D(v)}{\sqrt{2r_1}}\cos\left(\frac{\theta_1}{2}\right),\quad D_x=-\frac{K^D(v)}{\sqrt{2r_1}}\sin\left(\frac{\theta_1}{2}\right),\tag{37}$$

$$B_y=\frac{K^B(v)}{\sqrt{2r_1}}\cos\left(\frac{\theta_1}{2}\right),\quad B_x=-\frac{K^B(v)}{\sqrt{2r_1}}\sin\left(\frac{\theta_1}{2}\right),\tag{38}$$

$$q=\frac{\gamma_{11}e_{15}^2+\lambda_{11}h_{15}^2-2\beta_{11}e_{15}h_{15}}{c_{44}\left(\lambda_{11}\gamma_{11}-\beta_{11}^2\right)\sqrt{k}+\left(\gamma_{11}e_{15}^2+\lambda_{11}h_{15}^2-2\beta_{11}e_{15}h_{15}\right)\left(\sqrt{k}-1\right)},\tag{39}$$

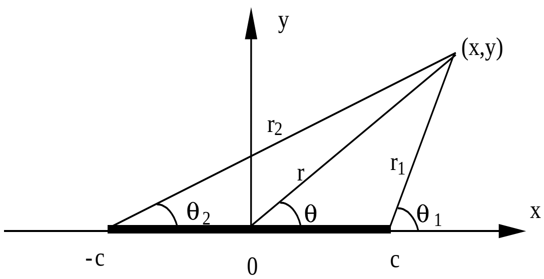

Fig. 2. Coordinates used to express solution.

where the polar coordinates r_1, θ_1 and $\tilde{r}_1$, $\tilde{\theta}_1$ are coordinates defined in Fig. 2, they are

$$r_1 = \sqrt{(x-c)^2 + y^2}, \quad \theta_1 = \tan^{-1}\left(\frac{y}{x-c}\right),$$
$$\tilde{r}_1 = \sqrt{(x-c)^2 + ky^2}, \quad \tilde{\theta}_1 = \tan^{-1}\left(\frac{\sqrt{k}y}{x-c}\right) \tag{40}$$

and $K^T(v)$, $K^D(v)$ and $K^B(v)$ are the dynamic stress intensity factor (DSIF), the dynamic electric displacement intensity factor (DEDIF), and the dynamic magnetic induction intensity factor (DMIIF), respectively; these field intensity factors can be defined as

$$\begin{aligned}
K^T(v) &= \lim_{x\to c^+} \sqrt{2(x-c)}\sigma_{zy}(x,0) = P_0\sqrt{c},\\
K^D(v) &= \lim_{x\to c^+} \sqrt{2(x-c)}D_y(x,0) = e_{15}R_1\sqrt{c}\\
&= \frac{e_{15}(\lambda_{11}\gamma_{11} - \beta_{11}^2)P_0\sqrt{c}}{c_{44}(\lambda_{11}\gamma_{11} - \beta_{11}^2)\sqrt{k} + (\gamma_{11}e_{15}^2 + \lambda_{11}h_{15}^2 - 2\beta_{11}e_{15}h_{15})(\sqrt{k}-1)},\\
K^B(v) &= \lim_{x\to c^+} \sqrt{2(x-c)}B_y(x,0) = h_{15}R_1\sqrt{c}\\
&= \frac{h_{15}(\lambda_{11}\gamma_{11} - \beta_{11}^2)P_0\sqrt{c}}{c_{44}(\lambda_{11}\gamma_{11} - \beta_{11}^2)\sqrt{k} + (\gamma_{11}e_{15}^2 + \lambda_{11}h_{15}^2 - 2\beta_{11}e_{15}h_{15})(\sqrt{k}-1)}.
\end{aligned} \tag{41}$$

For this particular problem, the stresses, electric displacements and magnetic inductions at the crack tip show the inverse square root singularities. It is clear that the DEDIF and DMIIF under the permeable crack condition are dependent on the speed of the moving crack and material constants.

Using the polar coordinate system (r_1, θ_1) defined near by the crack tip, the field intensity factors along the orientation θ_1 can be obtained as

$$K^T(v,\theta_1) = K^T(v)F(\theta_1), \quad K^D(v,\theta_1) = K^D(v)\cos\left(\frac{\theta_1}{2}\right),$$
$$K^B(v,\theta_1) = K^B(v)\cos\left(\frac{\theta_1}{2}\right), \tag{42}$$

where

$$F(\theta_1) = (1+q)\Omega(\theta_1)\left[\cos(\theta_1)\cos\left(\frac{\tilde{\theta}_1}{2}\right) + \frac{1}{\sqrt{k}}\sin(\theta_1)\sin\left(\frac{\tilde{\theta}_1}{2}\right)\right] - q\cos\left(\frac{\theta_1}{2}\right),$$
$$\Omega(\theta_1) = \sqrt{\frac{r_1}{\tilde{r}_1}} = \frac{1}{\sqrt[4]{(1-k)\cos^2(\theta_1) + k}}, \quad \tan(\tilde{\theta}_1) = \sqrt{k}\tan(\theta_1). \tag{43}$$

To illustrate the influence of the velocity of the moving crack on the DEDIF and DMIIF, a Mach number as the ratio of the velocity to the magnetoelectroelastic shear wave speed, $M = v/C$, is introduced. It is observed that from Eq. (41) that the magnitudes of $K^D(v)$ and $K^B(v)$, in the case of permeable condition will become infinity when

$$M = M_d = \sqrt{\frac{c_{44}(\lambda_{11}\gamma_{11} - \beta_{11}^2)\left[c_{44}(\lambda_{11}\gamma_{11} - \beta_{11}^2) + 2(\gamma_{11}e_{15}^2 + \lambda_{11}h_{15}^2 - 2\beta_{11}e_{15}h_{15})\right]}{\left[c_{44}(\lambda_{11}\gamma_{11} - \beta_{11}^2) + \gamma_{11}e_{15}^2 + \lambda_{11}h_{15}^2 - 2\beta_{11}e_{15}h_{15}\right]^2}}. \tag{44}$$

From Eq. (43), it can be seen that the function $F(\theta_1)$ is independent of the crack length $2c$. The crack length does not affect the distribution of the DSIF on the circumference. Therefore, analyzing the function $F(\theta_1)$ would provide a good model to understand the crack propagation orientation.

In the case of $v = 0$, $\beta_{11} = 0$, and $h_{15} = 0$, our results are exactly reduced to the static piezoelectric solutions, and are agreed with Zhang and Tong (1996). This shows that our solutions are correct and universal.

5. Discussions

We will consider a transversely isotropic material exhibiting full coupling between elastic, electric and magnetic fields, with unique axis along x_3 direction. The independent material constants are the elastic constants, piezoelectric constants, piezomagnetic constants, dielectric constants, magnetic constants and magnetoelectric constants. This is the general situation, and for a particular material, some of the coupling coefficients may be zero. The material constants we used are given by Li (2000) as follows:

$$\begin{aligned} &c_{44} = 4.53 \times 10^{10}\ (\mathrm{N/m^2}), \quad e_{15} = 11.6\ (\mathrm{C/m^2}), \quad h_{15} = 550\ \mathrm{N/Am}, \\ &\lambda_{11} = 0.8 \times 10^{-10}\ (\mathrm{C^2/Nm^2}), \quad \gamma_{11} = -5.9 \times 10^{-4}\ (\mathrm{Ns^2/C^2}), \\ &\beta_{11} = 0.5 \times 10^{-11}\ (\mathrm{Ns/VC}). \end{aligned} \tag{45}$$

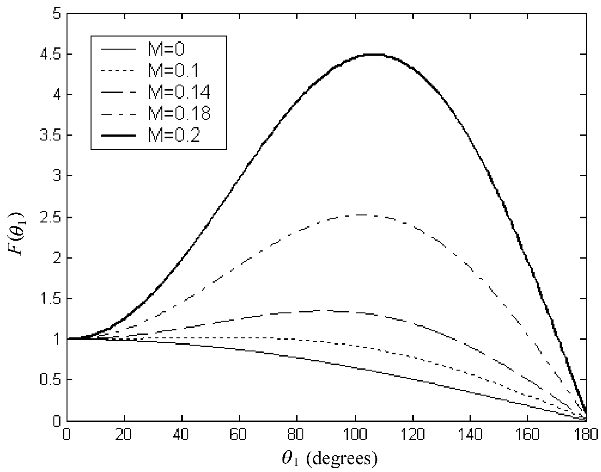

Fig. 3. $F(\theta_1)$ versus θ_1 when $0 \leqslant M < M_d$.

Table 1
Values of $F(\theta_1)$ against M and maximum value $F(\theta_b)$

M	θ_1					
	0°	30°	60°	90°	120°	$F(\theta_b)$
0.1 ($\theta_b =$ 53°)	1.0000	1.0155	1.0192	0.9580	0.7661	1.0209
0.12 ($\theta_b =$ 76°)	1.0000	1.0391	1.1124	1.1111	0.9285	1.1288
0.14 ($\theta_b =$ 88°)	1.0000	1.0813	1.2551	1.3462	1.1779	1.3467
0.16 ($\theta_b =$ 96°)	1.0000	1.1505	1.4901	1.7339	1.5891	1.7431
0.18 ($\theta_b =$ 102°)	1.0000	1.2803	1.9317	2.4642	2.3636	2.5205
0.19 ($\theta_b =$ 104°)	1.0000	1.3979	2.3326	3.1280	3.0677	3.2369
0.20 ($\theta_b =$ 105°)	1.0000	1.6015	3.0275	4.2795	4.2889	4.4860
0.21 ($\theta_b =$ 107°)	1.0000	2.0381	4.5186	6.7516	6.9108	7.1766
0.22 ($\theta_b =$ 108°)	1.0000	3.6332	9.9688	15.6589	16.4979	17.0289
0.225 ($\theta_b =$ 109°)	1.0000	9.1235	28.7313	46.9146	49.5066	50.9679

By analyzing the extreme value of function $F(\theta_1)$, we find the Mach number exists a critical value when $M_{c1} = 0.087$. While $M \leqslant M_{c1}$ and $0° \leqslant \theta_1 \leqslant 180°$, $F(\theta_1)$ monotonically decreases with increase of θ_1, see Fig. 3. The maximum value of the DSIF $K^T(v,\theta_1)$ occurs at the crack axis $\theta_1 = 0°$, this means that the crack has a tendency to propagate along its original plane when the criterion of the maximum tensile stress is used.

For the case of $M_{c1} < M < M_d$ and $0° \leqslant \theta_1 \leqslant 180°$, $F(\theta_1)$ increases with increase of θ_1 at first and then decreases after it reaches a certain peak value. It is shown that the orientation of the maximum DSIF makes a branch angle of θ_b with the crack axis, and the higher crack propagation speed, the bigger branch angle. This conclusion will be in agreement with that obtained by Hou et al. (2001), Kwon and Lee (2001) and Kwon et al. (2002) when our solution reduce to piezoelectric material case.

When M varies from 0.21 to 0.225 (while $q \to \infty$, $M \to M_d = 0.2276$), θ_b approximately ranges from 107° to 109°. Some results are listed in Table 1.

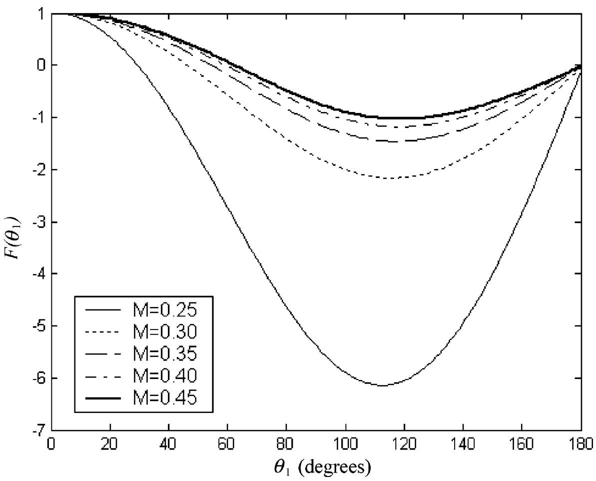

Fig. 4. $F(\theta_1)$ versus θ_1 when $M_d < M < M_{c2}$.

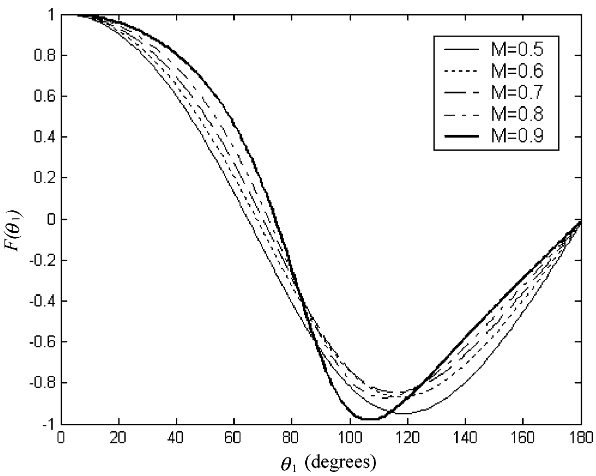

Fig. 5. $F(\theta_1)$ versus θ_1 when $M_{c2} < M < 1$.

For the case of $M > M_d$ and $0° \leqslant \theta_1 \leqslant 180°$, the Mach number also exists a critical value when $M_{c2} = 0.476$. From Fig. 4, it can be seen that the maximum magnitudes of $F(\theta_1)$ is greater than 1 at an angle $\theta_1 \neq 0°$ when $M_d < M < M_{c2}$, this means that the crack will deviate from its original plane. While at higher crack velocity, the maximum magnitudes of $F(\theta_1)$ is always 1 at angle $\theta_1 = 0°$ when $M_{c2} < M < 1$, this means that the crack will propagate along its original plane, see Fig. 5.

Fig. 6 shows the variations of the normalized DEDIF $K^{D^*} = 10^9 K^D(v)/(P_0\sqrt{c})$ versus M. The influence of the speed of the moving crack on the normalized DMIIF $K^{B^*} = 10^7 K^B(v)/(P_0\sqrt{c})$ was shown in Fig. 7.

For the case that $0 < M < M_d$, the DEDIF and the DMIIF gradually enlarge with the increase of crack speed, and will increase rapidly and verge on positive infinity when M verges on M_d.

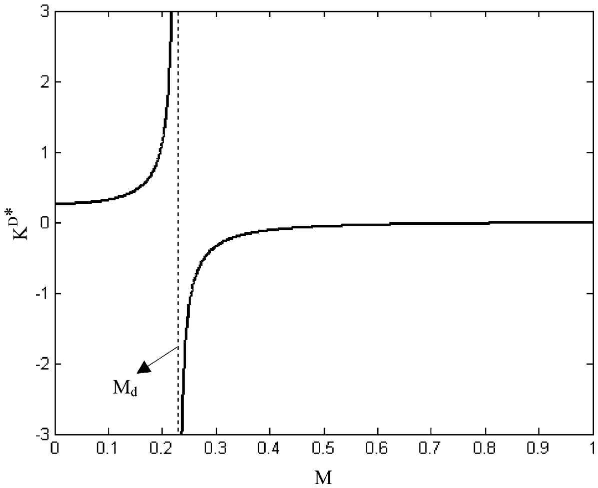

Fig. 6. The normalized DEDIF $K^{D^*} = 10^9 K^D(v)/(P_0\sqrt{c})$versus M.

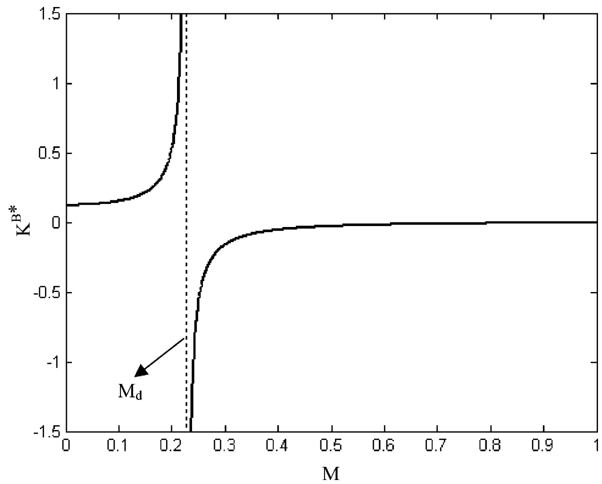

Fig. 7. The normalized DMIIF $K^{B^*} = 10^7 K^B(v)/(P_0\sqrt{c})$ versus M.

For the case where $M_d < M \leqslant 1$, the DEDIF and the DMIIF gradually enlarge from $-\infty$ with the increase of crack speed to certain values of $K^{D^*}(1)$ and $K^{B^*}(1)$ respectively. The values of $K^{D^*}(1)$ and $K^{B^*}(1)$ are

$$K^{D^*}(1) = \frac{10^9 e_{15}(\lambda_{11}\gamma_{11} - \beta_{11}^2)}{2\beta_{11}e_{15}h_{15} - \gamma_{11}e_{15}^2 - \lambda_{11}h_{15}^2}, \quad K^{B^*}(1) = \frac{10^7 h_{15}(\lambda_{11}\gamma_{11} - \beta_{11}^2)}{2\beta_{11}e_{15}h_{15} - \gamma_{11}e_{15}^2 - \lambda_{11}h_{15}^2}. \tag{46}$$

6. Conclusions

The magnetoelectroelastic problem of a constant moving crack in an orthotropic magnetoelectroelastic material under the combined anti-plane mechanical shear and in-plane electrical and magnetic loadings has been analyzed for permeable crack condition by integral transform approach. Closed-form solution of the field variables and the field intensity factors are derived. The stresses, electric displacements and magnetic inductions at the crack tip exhibit the inverse square root singularities. The DEDIF and DMIIF under the permeable crack condition are dependent on the speed of the moving crack and material constants. When the velocity of the moving crack is less than M_{c1} or higher than M_{c2}, the crack will propagate along its original plane; while in the range of $M_{c1} < M < M_{c2}$, the propagation of the crack possibly brings about the branch phenomena in magnetoelectroelastic media.

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